JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 5, September–October 1995

Technical Comments

Comment on "Optimal Cooperative Power-Limited Rendezvous Between Neighboring Circular Orbits"

Yurij P. Ulybyshev*
NPO "Energia," Kaliningrad, Moscow Region 141070, Russia

REFERENCE 1 investigates the solution of the problem of determining optimal cooperative power limited rendezvous between neighboring circular orbits using linearized equations of the relative motion developed by Wheelon[2] and independently by Clohessy and Wiltshire.[3] In terms of the radial (x), circumferential (y), and out-of-plane (z) displacements, the Clohessy–Wiltshire (CW) equations are

$$\begin{aligned} \ddot{x}(t) &= 3n^2x(t) + 2n\dot{y}(t) + \Gamma_x \\ \ddot{y}(t) &= -2n\dot{x}(t) + \Gamma_y \\ \ddot{z}(t) &= -n^2z(t) + \Gamma_z \end{aligned} \tag{1}$$

where Γ_x, Γ_y, and Γ_z the components of the thrust acceleration vector, and the constant angular velocity of the spacecraft is indicated by the scalar n.

The solution to Eqs. (1) for the motion due to the optimal thrust acceleration coinciding with the primer vector $\boldsymbol{P}^T = (p_x, p_y, p_z)$ developed by Lawden[4] can be rewritten as[1]

$$\boldsymbol{Y}_1(t) = \boldsymbol{\Phi}_{rr}(t - t_0)\boldsymbol{Y}_1(t_0) + \boldsymbol{\Phi}_{rp}(t - t_0)\boldsymbol{Y}_2(t_0) \tag{2}$$

where $\boldsymbol{Y}_1^T = (x, y, z, \dot{x}, \dot{y}, \dot{z})$ and $\boldsymbol{Y}_2^T = (\boldsymbol{P}^T, \dot{\boldsymbol{P}}^T)$, and $\boldsymbol{\Phi}_{rr}$ and $\boldsymbol{\Phi}_{rp}$ are the state transition matrices.[1]

Similar studies of the CW equations for optimal rendezvous have been conducted by Marinescu[5] and Lembeck and Prussing.[6] The solution of Eqs. (1) by Lembeck and Prussing[6] can be obtained as

$$\boldsymbol{Y}_1(t) = \boldsymbol{\Phi}_{rr}(t - t_0)\boldsymbol{Y}_1(t_0) + \boldsymbol{D}(t - t_0)\boldsymbol{q} \tag{3}$$

where $\boldsymbol{q}^T = (\alpha, \beta, \gamma, \delta, \varepsilon, \zeta)$ is the vector containing the unknown constants of the primer vector, and $\boldsymbol{D}$ is the state transition matrix.[6] The optimal thrust acceleration is[6]

$$\Gamma = \begin{bmatrix} \alpha\cos t + \beta\sin t + \gamma \\ 2\beta\cos t - 2\alpha\sin t - 1.5\gamma t + \delta \\ \varepsilon\cos t + \zeta\sin t \end{bmatrix} \tag{4}$$

Solutions (2) and (3) are conceptually identical. One can determine that there exists a linear relation between $\boldsymbol{Y}_2^T(t_0) = (\boldsymbol{P}_0^T, \dot{\boldsymbol{P}}_0^T)$ and $\boldsymbol{q}$. The corresponding solution (2) for optimal thrust acceleration can be described as

$$\Gamma = \begin{bmatrix} -(3p_{x0} + 2\dot{p}_{y0})\cos t + \dot{p}_{x0}\sin t + 4p_{x0} + 2\dot{p}_{y0} \\ 2\dot{p}_{x0}\cos t + (6p_{x0} + 4\dot{p}_{y0})\sin t \\ -1.5(4p_{x0} + 2\dot{p}_{y0})t + p_{y0} - 2\dot{p}_{x0} \\ p_{z0}\cos t + \dot{p}_{z0}\sin t \end{bmatrix} \tag{5}$$

With the aid of the transformation

$$\boldsymbol{q} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \\ \zeta \end{bmatrix} = \boldsymbol{Q} \begin{bmatrix} p_{x0} \\ p_{y0} \\ p_{z0} \\ \dot{p}_{x0} \\ \dot{p}_{y0} \\ \dot{p}_{z0} \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{x0} \\ p_{y0} \\ p_{z0} \\ \dot{p}_{x0} \\ \dot{p}_{y0} \\ \dot{p}_{z0} \end{bmatrix} \tag{6}$$

Eq. (4) may be rewritten as Eq. (5). It is also true that

$$\boldsymbol{\Phi}_{rp}(t - t_0) = \boldsymbol{D}(t - t_0)\boldsymbol{Q} \tag{7}$$

The inverse transformation from (6) is described by Lembeck and Prussing in Ref. 6 [Eq. (26)].

To our knowledge,[7] the first solution to Eqs. (1) due to the optimal thrust acceleration Eq. (4) is given by Marinescy.[5]

Reference 1 contains errata. The fourth term in the expression for K_{111} (see Appendix, p. 1053) should be preceded by a positive sign:

$$K_{111} = (77/2)T - (27/4)\sin(2T) + 12T^3 + 72T\cos(T) - 96\sin(T)$$

The expressions (see Appendix, p. 1053)

$$K_{223} = (1/2)\sin^2(T)$$

$$K_{233}(T) = K_{223}(T) = \cdots = 0$$

should be

$$K_{233} = (1/2)\sin^2(T)$$

$$K_{213}(T) = K_{223}(T) = \cdots = 0$$

References

[1]Coverstone-Carroll, V., and Prussing, J. E., "Optimal Cooperative Power-Limited Rendezvous Between Neighboring Circular Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 6, 1993, pp. 1045–1054.

[2]Wheelon, A. D., "Midcourse and Terminal Guidance," *Space Technology*, Wiley, New York, 1959, Chap. 26.

[3]Clohessy, W. H., and Wiltshire, R. S., "Terminal Guidance Systems for Satellite Rendezvous," *Journal of the Aerospace Sciences*, Vol. 27, Sept. 1960, pp. 653–658, 674.

[4]Lawden, D. F., *Optimal Trajectories for Space Navigation*, Butterworths, London, 1963, Chap. 3.

[5]Marinescu, A., "Optimal Low-Thrust Orbital Rendezvous," *Journal of Spacecraft and Rockets*, Vol. 13, No. 7, 1976, pp. 385–392.

[6]Lembeck, C. A., and Prussing, J. E., "Optimal Impulsive Intercept with Low-Thrust Rendezvous Return," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3, 1993, pp. 426–433.

[7]Ulybyshev, Y. P., "Comment on 'Optimal Impulsive Intercept with Low-Thrust Rendezvous Return,'" *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 6, 1994, p. 1392.

Received April 9, 1994; accepted for publication Nov. 28, 1994.

*Group Leader, Space Ballistics Department.